Comment on "Signal-to-noise ratio gain in neuronal systems"

Igor A. Khovanov[1,2] and Peter V. E. McClintock[2]
[1]*Department of Physics, Saratov State University, Astrakhanskaya Street 83, 410071 Saratov, Russia*
[2]*Department of Physics, Lancaster University, Lancaster LA1 4YW, United Kingdom*
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We discuss the gain in signal-to-noise ratio (SNR) recently reported by Liu *et al.* [Phys. Rev. E **63**, 051912 (2001)] in the Hodgkin-Huxley neuronal model. We show first that the possibility of signal-to-noise ratio enhancements can be checked by consideration of the statistical characteristics of switching between the system states, and we examine how the SNR depends on the shape of a periodic signal. Second, we attempt to verify the SNR gain reported by Liu *et al.*: based on spectral calculations and analyses of switching statistics, we are unable to find any SNR gain in the Hodgkin-Huxley model.

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Liu *et al.* have reported [1] noise-induced absolute enhancements of the signal-to-noise ratio (SNR) in the Hodgkin-Huxley model of a neuron. Solving the equations numerically, they found that the output SNR could exceed the input SNR in the following cases: (a) a single neuron forced by harmonic signal and coloured noise; (b) a single neuron forced by pulse signal and colored noise; and (c) a neural network forced by a common harmonic signal and independent colored noise sources.

In this Comment we present arguments and numerical results demonstrating that, for the same parameters as Ref. [1], such SNR improvements do not occur for either case (a) or case (b).

In apparently demonstrating the possibility of an SNR enhancement, Liu *et al.* [1] have relied on a spectral calculation of the input and output neuron signals, obtained through numerical solution of the Langevin equations. Previous studies [2,3] of the stochastic resonance (SR) effect have shown that numerical calculations of input and output SNR values are very readily prone to error. They have also demonstrated [2] that, at least in the linear response limit, the SNR of a harmonic signal *cannot* be enhanced by passage through a stochastic resonator.

A noise-induced absolute enhancement of the SNR is thus, to say the least, unusual. There are only two known cases where it has been reliably established. One is the level crossing detector (LCD) trigger system [4], and the other is for the passage of a square wave through a Schmidt trigger system [5,6]. In each of these cases there are simple physical arguments as to how the effect arises, and a mass of corroboratory evidence as to its reality. For the more complex Hodgkin-Huxley system [1], on the other hand, there are *only* the numerical simulations. Methods of checking their validity are therefore much to be desired. One way of doing so is through consideration of the theoretical background (although rigorous theoretical arguments exist only for the specific cases of the LCD-trigger [4] and a two-state system with a static nonlinearity [7]). Another obvious check is through independent numerical simulations. In what follows, we adopt both of these approaches.

We first consider the occurrence of absolute SNR enhancements in the Schmidt trigger, and note how the signature of this effect is exhibited in the hopping statistics. We then model the Hodgkin-Huxley system to see if a similar signature arises. We also simulate the full system and make direct comparisons between the input and output SNR to see if the latter becomes larger that the former.

In Refs. [5,6], SNR enhancement was investigated via an analysis of the statistical characteristics of switching events between states. We now consider some of these results [6] in more detail. We have examined the transmission of a periodic signal and colored noise through a Schmidt trigger on the basis of an analog electronic experiment. We have considered two types of periodic signals: harmonic signal and rectangular, which we will refer to as a *pulse signal*. To avoid aliasing, the experimental data were filtered by low-pass analog filters. The data were then digitized with an analog-digital converter card in a computer, where they were then used to calculate the power spectrum and to build a residence time [8] and phase [9] distributions. The power

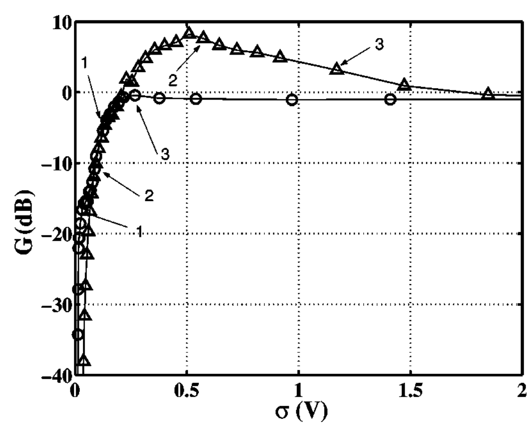

FIG. 1. Experimental results for the Schmidt trigger. The difference G between the output and input SNR R is plotted as a function of noise intensity σ for harmonic (○) and for pulse periodic (△) signals. The threshold of the Schmidt trigger, $U_t = 0.47$ V; the amplitude of the periodic signal is 0.41 V; the frequency of the periodic signal is $2\pi(400)$ Hz; and the cutoff frequency of the noise is 100 kHz. The values of G and noise intensity σ for which residence time and phase distributions are shown in Figs. 2–4 are indicated by arrows.

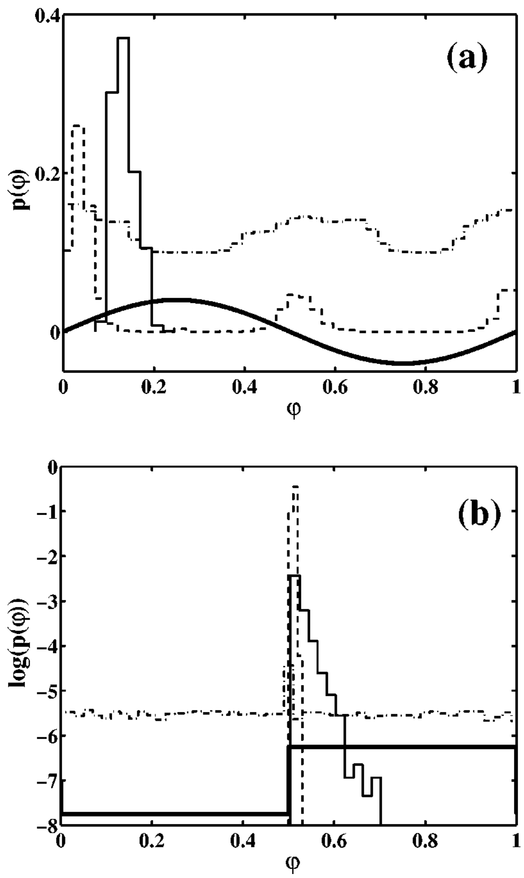

FIG. 2. Experimental results for the Schmidt trigger. Its phase distributions $p(\varphi)$ are shown for (a) harmonic and (b) pulse periodic signals. The different lines correspond to the arrows in Fig. 1. The thin full line corresponds to arrow 1, the dashed line to arrow 2, and the dash-dotted line to arrow 3. The values of phase φ are normalized by 2π. The bold full lines indicate the shape of signals.

spectrum was calculated by fast Fourier transform. The results are presented in Figs. 1–3.

In Fig. 1 the difference between output and input SNR [10], $G=R_o-R_i$, is plotted as a function of noise amplitude σ for pulse and harmonic signals; note that G is equivalent to the quantity g_u in Ref. [1]. For the pulse signal there is an interval of noise amplitude within which the output SNR R_o exceeds the input value R_i. The residence time and phase distributions for different noise amplitudes (see caption of Fig. 1) are shown in Figs. 2,3. For the harmonic signal, the widths of the phase distribution [Fig. 2(a)] and the residence time distribution [Figs. 3(a)-3(c)] are increasing with noise intensity, and there is no improvement of SNR. For the pulse signal, within the range of noise amplitude where output SNR exceeds the input value, the distribution widths [Figs. 2(b) and 3(d)-3(f)] decrease with noise intensity. The residence time distribution [Figs. 3(d)-3(f)] has peaks at multiples of the signal period when an improvement in SNR exists. Thus, based on the behaviors of residence times and phase distributions, we can suggest an additional criterion for the existence of a positive G: if there is SNR gain, then the residence time distribution has one peak arising at multiples of the signal period, and the widths of both the residence time and phase distribution decrease with increasing noise intensity. The conclusion is correct if we consider hopping dynamics only. This criterion can be applied to check the results reported in Ref. [1], which involved precisely this kind of dynamics.

Note that, just as for the Schmidt trigger, we have observed an association between an improvement in SNR and an evolution of the hopping statistic distribution for the LCD trigger and an overdamped bistable oscillator forced by large, but subthreshold, pulses and noise. In all the cases the systems are in the strongly nonlinear regime [2,3,5], and the mechanism for improvement of the SNR consists of a nonlinear transformation of additive sum of pulse signal and noise to qualitatively different hopping process. The positive G phenomenon depends crucially on the pulse *shape*, and it is not observed for harmonic subthreshold signals [5,6].

To illustrate the truth of the latter statement, we consider the influence of a shape of periodic signal on SNR for the case of a Schmitt trigger driven by noise and the signal $x(t)$ that is generated by a Van der Pol self-oscillator,

$$y(t+h)=\mathrm{sgn}[U_t y(t)-ax(t)/b-\xi(t)],$$
$$\ddot{x}-\epsilon(1-x^2)\dot{x}+\omega_0^2 x=0. \tag{1}$$

Here $y(t+h)$ and $y(t)$ are the Schmidt trigger outputs at time moments $(t+h)$ and t, respectively, h is the time step that defines the trigger relaxation time, $U_t=0.1$ is the trigger threshold, $\xi(t)$ is colored noise of intensity σ, cutoff frequency $f_c=100$, a is the amplitude (maximum deviation from zero level) of the signal, $x(t)$ is the Van der Pol oscillator output, b is its amplitude, ϵ is the nonlinearity parameter of the oscillator, and ω_0 is the parameter that defines its natural oscillation frequency. The parameter ω_0 is chosen to keep the oscillation period equal to $T=2\pi$. The shape of the generated signal x depends on ϵ and approaches a rectangular shape as ϵ increases. To characterize the shape of the signal x, we introduce the quality S:

$$S=\frac{\int_0^T |x(t)|/b}{T}.$$

S is equal to 1 for a pulse signal and $S=1/\pi\approx 0.6366$ for a harmonic signal.

The dependences of G on the noise amplitude σ are displayed in Fig. 4 for different values of ϵ. It can be seen that for the signal shapes that are harmonic, or close to harmonic, SNR improvement does not occur: G only increases above zero for larger values of ϵ (and for the pulse signal, shown for comparison); and SNR improvement takes place only for certain shapes of signals, which are similar to the (rectangular) pulse signal shape.

Of course, as the authors of Ref. [1] have correctly noted, the Hodgkin-Huxley model is quite different from all the

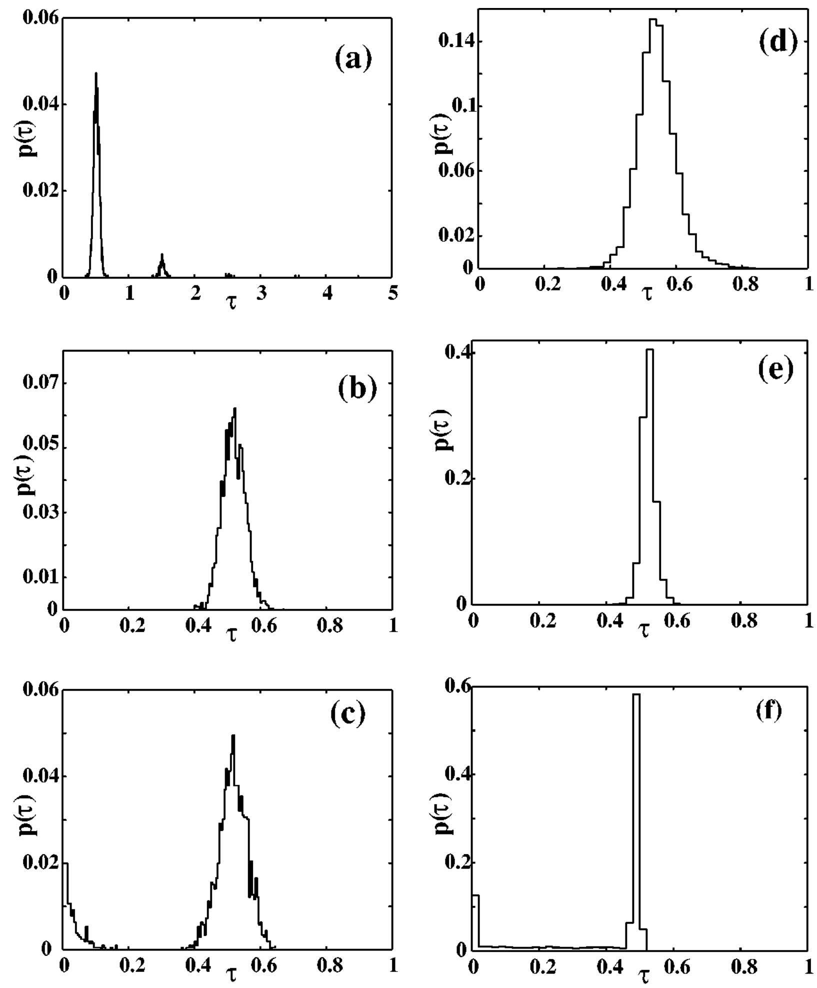

FIG. 3. Experimental results for the Schmidt trigger driven by a harmonic signal (left column) and pulse periodic signal (right column). Residence time distributions $p(\tau)$ shown for three different noise intensities correspond to the arrows in Fig. 1, (a) and (d) correspond to arrow 1, (b) and (e) to arrow 2, and (c) and (f) to arrow 3. The values of time are normalized by the period of the harmonic signal.

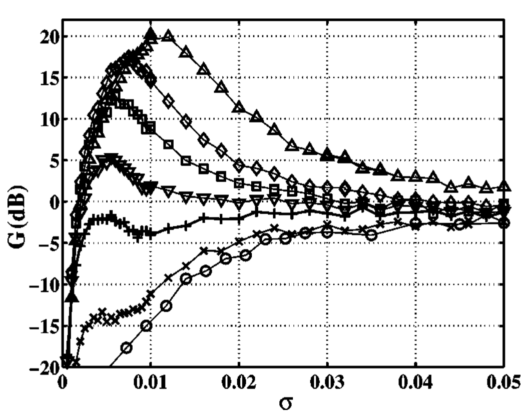

FIG. 4. The results of numerical simulations of system (1). The difference in SNR R between input and output, G, is plotted as a function of noise intensity σ for different *shapes* of the periodic signal: harmonic signal (○) with $S=2/\pi\approx 0.6366$; pulse signal (△) with $S=1$; parameter $\epsilon=1$ (×) with $S\approx 0.6493$; $\epsilon=5$ (+) with $S\approx 0.7455$; $\epsilon=10$ (▽) with $S\approx 0.7862$; $\epsilon=25$ (□) with $S\approx 0.9265$; and $\epsilon=100$ (◇) with $S\approx 0.9822$.

systems for which the positive G has been reported previously. The main difference is the presence in the Hodgkin-Huxley model of several time scales: the relaxation time to the stable state, and scales which define the intra-well dynamics. This is in contrast to trigger systems and the over-damped bistable oscillator, which are characterized only by a relaxation time scale. Liu *et al.* [1] suggest that their results (i.e., the absolute enhancement of the SNR) may be attributed to the presence of intrawell dynamics, in addition to interwell dynamics. This suggestion is apparently inconsistent with the results of earlier investigations [11,12], showing that the SNR and amplification are both *decreased* by increasing complexity of the intrawell dynamics.

In the context of earlier work, therefore, the results of Ref. [1] for the Hodgkin-Huxley model must be regarded as quite unexpected, especially in cases (a) and (b). We have tried to reproduce their results using the same parameters and a different numerical scheme for integration of the Langevin equations. We have also chosen an integration time step that is four times smaller than in Ref. [1] (for the same noise

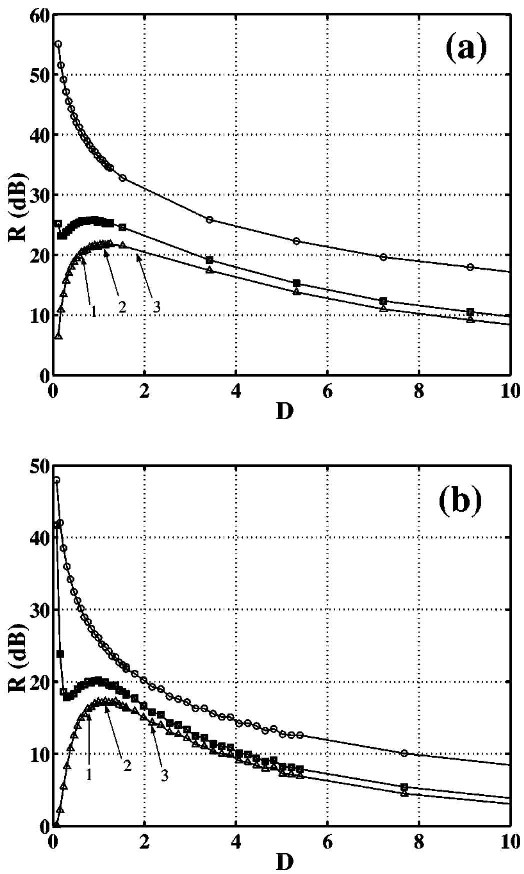

FIG. 5. Results of numerical simulations for the Hodgkin-Huxley model. The SNR R is plotted as a function of noise intensity D (a) for a harmonic signal and (b) for a pulse signal. The input R_i ($\bigcirc$) is compared in each case with the output R_o: for full dynamics ($\square$); and for pulse dynamics ($\triangle$). The values of R and noise intensity D for which residence time distributions and phase distributions are shown in Figs. 6 and 7 are indicated by arrows.

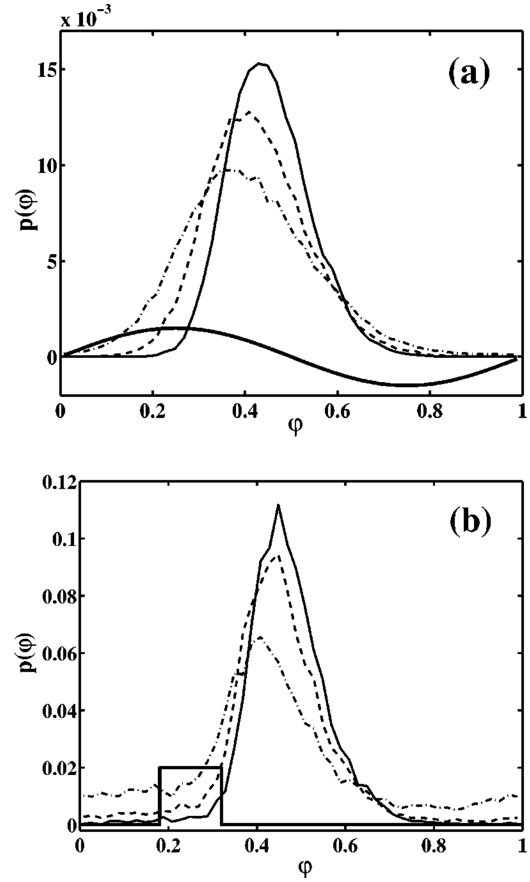

FIG. 6. Phase disributions $p(\varphi)$ for the Hodgkin-Huxley model driven by (a) harmonic and (b) pulse periodic signals. The different lines relate to different noise intensities, corresponding to the arrows in Fig. 5; the thin full line corresponds to arrow 1, the dashed line to arrow 2, and the dash-dotted line to arrow 3. The values of phase are normalised by 2π. The bold full curves represent the shape of signals.

correlation time), in order to be confident that we have avoided possible aliasing effects [13]. The numerical scheme has been exhaustively tested and approved for use in a very wide range of problems [14]. In addition to the power spectrum, we have also calculated time statistical distributions of spikes (switching between states) of the neuron.

The results of our numerical simulations are presented in Figs. 5–7. We used the same parameters as Liu *et al.* [1] [see the caption of Fig. 1(a) in Ref. [1] for parameters of the harmonic signal, and the caption of Fig. 3 in Ref. [1] for parameters of the pulse signal]. We have calculated the SNR for the full dynamics (without transformation to a two-state dynamics) of the neuron voltage potential $V(t)$, and also for the spike dynamics of $V(t)$, which was obtained by converting $V(t)$ into a sequence of pulses with the same amplitude and duration as in Ref. [1]. Note that in Ref. [1], only the last type of dynamics was examined. It can be seen (Fig. 5) that, for both harmonic and pulse signals, the output SNRs behave like those in Ref. [1] (see Figs. 1(a) and 3 on Ref. [1]) only in the region of the maximum. For larger noise intensity D, the output SNRs show quite different behaviors. The input SNRs are different for all the values of noise amplitude. In contrast to Ref. [1], it is clear that the present data show no SNR improvement for either harmonic or pulse signals.

The form of the time statistical distributions (Figs. 6 and 7) again demonstrate the absence of a positive G. Indeed, the width of circle distributions increases with noise intensity (Fig. 6), and the residence time distributions have several peaks separated by periods of the signal (Fig. 7).

We have not attempted to reproduce the results reported [1] for case (c). However, especially given that Liu *et al.* used the same numerical algorithm as for the single-neuron cases (a) and (b), the results of case (c) must be in question and deserve to be checked.

In summary, using the same parameters as the authors of Ref. [1], we can find no evidence to support the reported observation of absolute SNR enhancements in the Hodgkin-Huxley model of a neuron. Finally, we stress the close connection that exists between SNR enhancements and hopping processes: statistical measures of hopping dynamics can be used to check the correctness of spectral calculations. Information flow in neuron dynamics can be characterized by statistical measures of the hopping process, whereas the SNR cannot be used for this purpose [15].

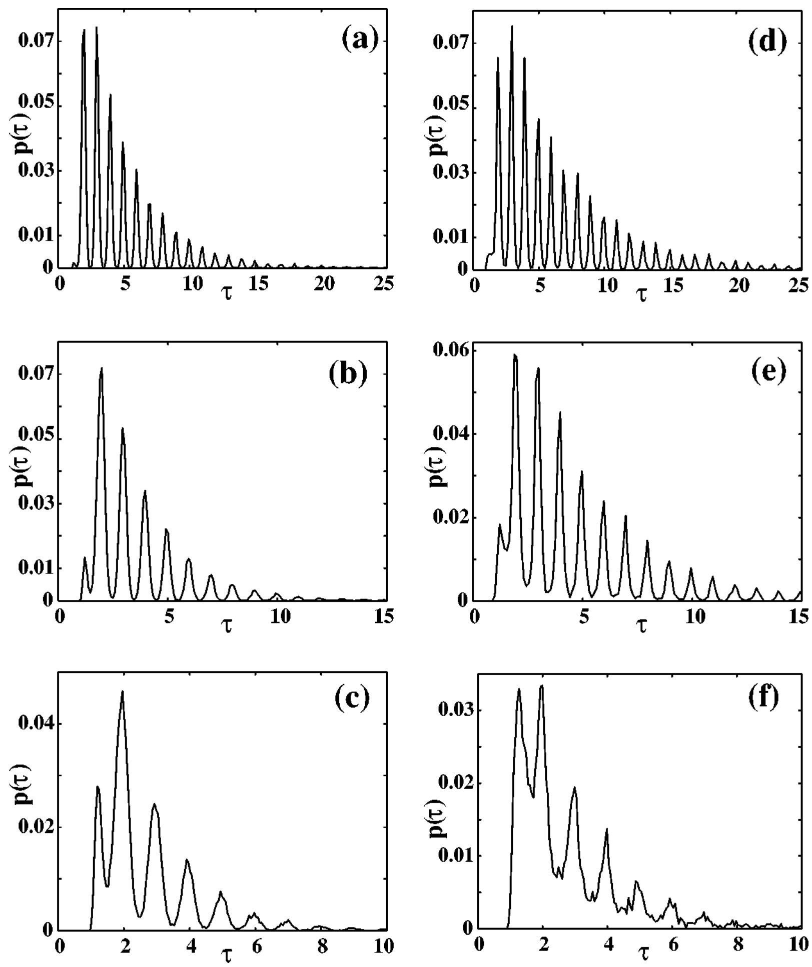

FIG. 7. Residence time distributions $p(\tau)$ are shown for the Hodgkin-Huxley model driven by a harmonic signal (left column) and pulse periodic signal (right column). The different figures are for different noise intensities corresponding to the arrows in Fig. 5; (a) and (d) correspond to arrow 1, (b) and (e) to arrow 2, and (c) and (f) to arrow 3. The values of time are normalized by the period of the signal.

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[1] F. Liu, Y. Yu, and W. Wang, Phys. Rev. E **63**, 051912 (2001).

[2] M.I. Dykman and P.V.E. McClintock, Nature (London) **391**, 344 (1998).

[3] P. Hanggi, M.E. Inchiosa, D. Fogliatti, and A.R. Bulsara, Phys. Rev. E **62**, 6155 (2000).

[4] L. Kiss, in *Chaotic, Fractal, and Non-linear Signal Processing*, edited by R.A. Katz, AIP Conf. Proc. 375 (AIP, Woodbury, New York, 1996), pp. 382–395; K. Loerincz, Z. Gingl, and L.B. Kiss, Phys. Lett. A **224**, 63 (1996).

[5] I.A. Khovanov and V.S. Anishchenko, in *Applied Nonlinear Dynamics and Stochastic Systems Near the Millennium*, edited by J.B. Kadtke and A. Bulsara, AIP Conf. Proc. 411 (AIP, Woodbury, New York, 1997).

[6] I.A. Khovanov, Ph.D. thesis, Saratov State University, 1997.

[7] F. Chapeau-Blondeau and X. Godivier, Phys. Rev. E **55**, 1478 (1997).

[8] T. Zhou, F. Moss, and P. Jung, Phys. Rev. A **42**, 3161 (1990).

[9] A.S. French, A.V. Holden, and R.B. Stein, Kybernetik **11**, 15 (1972).

[10] The SNR for the pulse signal in Refs. [6,7] (like that in Ref. [1]) is defined, by analogy with the conventional definition used for a harmonic signal, as the logarithm of the ratio between the intensity of the δ peak in the power spectrum at the pulse repetition frequency to the intensity of the noisy background at the same frequency.

[11] V.S. Anishchenko, I.A. Khovanov, and B.V. Shulgin, in *Chaotic, Fractal, and Non-linear Signal Processing*, edited by R.A. Katz, AIP Conf. Proc. 375 (AIP, Woodbury, New York, 1996), pp. 363–381.

[12] L. Alfonsi, L. Gammaitoni, S. Santucci, and A.R. Bulsara,

Phys. Rev. E **62**, 299 (2000).

[13] As an additional check of the robustness of our SNR calculation, we replaced the Hodgkin-Huxley model by an overdamped linear oscillator, and calculated the input and output SNR for several noise intensities. We obtained an equality of the input and output SNR values to an accuracy better than 0.3 dB.

[14] R. Mannella, in *Stochastic Processes in Physics, Chemistry, and Biology,* edited J.A. Freund and T. Poschel, Lecture Notes in Physics, Vol. 557 (Springer-Verlag, Berlin, 2000), pp. 353–362.

[15] W.C. Robinson, J. Rung, A.R. Bulsara, and M.E. Inchiosa, Phys. Rev. E **63**, 011107 (2001).